FUKIEN SECONDARY SCHOOL
S5 First Term Examination (2020-2021)
Mathematics Extended Part Module 1
(2 hours)

Date: 5th January 2021 Name:____________________
Time: 10:30 a.m. - 12:30 p.m. Class: ________ No.: _______

Instructions to students:

1. This paper consists of Section A (75 marks) and Section B(25 marks).

2. Attempt ALL questions. Write your answers on the single-lined paper.

3. Unless otherwise specified, show your workings clearly.

4. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

5. The diagrams in this paper are not necessarily drawn to scale.

Section A (75 marks)

1. Let X and Y be two events. Suppose that $P(X)=0.4$, $P(Y)=0.33$ and $P(Y' \mid X')=0.7$, where X' and Y' are the complementary events of X and Y respectively.
   (a) Find $P(X' \cap Y')$.
   (b) Find $P(X \cup Y)$.
   (c) Are X and Y independent? Explain your answer.

(6 marks)

2. 20% of the students in a dancing school are adults, where 70% of them are females. 30% of the students in the school are males. A student is selected from the school randomly.
   (a) Find the probability that the student is a female who is not an adult.
   (b) Find the probability that the student is not an adult given that the student is a female.
   (c) Find the probability that the student is a male given that the student is not an adult.

(6 marks)

3. In a national football team, 10 players come from the football clubs in the English league, 5 from those in the Spanish league and 8 from those in the German league. 3 of them are goalkeepers and they all come from the football clubs in the English league. 11 players including one goalkeeper are randomly selected to form a starting squad. Let X be the number of players in the starting squad who come from the football clubs in the English league.
   (a) Represent the probability distribution of X by a table.
   (b) Find the probability that more than half of the players in the starting squad come from the football clubs in the English league.

(6 marks)

4. X is a random variable with $\mathrm{E}(X)=1.6$ and $\mathrm{Var}(X)=2.25$. If $Y=aX+b$ where a and b are integers, then the expectation and variance of Y are 3.4 and 36 respectively.
   (a) Find the values of a and b.
   (b) Find $\mathrm{Var}(bX-a)$ and $\mathrm{E}[(bX-a)^2]$.

(6 marks)

5. The table below shows the probability distribution of a discrete random variable X, where k is a constant.

x	6	8	k	16	20
$P(X=x)$	0.2	0.2	0.1	0.2	0.3

It is given that $\mathrm{Var}(X) = 3\mathrm{E}(X) - 9.05$.
   (a) Find the value of k.
   (b) Find $\mathrm{E}(2X-4)$.

(6 marks)

6. It is given that m and n are constants. Let X and Y be two independent discrete random variables with their respective probability distributions shown as follows:

$P(X=x)=\dfrac{2x-1}{28}$ for $x=m, n, 5, 7$

$P(Y=y)=\dfrac{y}{20}$ for $y=2, 3, 4, 5, 6$

Suppose that $\mathrm{E}(X)=\dfrac{38}{7}$ and $m<n$.

(a) Find the values of m and n.

(b) Let A be the event that $X=Y$, and B be the event that $Y>3$.

(i) Find $P(A\cap B)$.

(ii) Are A and B independent? Explain your answer.

(7 marks)

7. It is given that $Y\sim \mathrm{Po}(\lambda)$ and $[P(Y=2)]^2=3P(Y=4)$.

(a) Find the value of λ.

(b) Find $P(Y>0)$.

(c) Joseph claims that $P(Y$ is odd) is less than 0.4. Do you agree? Justify your answer.

(6 marks)

8. In a school, 45% of the students wear glasses. There are 3 after-school tutorial classes and each class has 5 students.

(a) Find the probability that among all the students joining the after-school tutorial classes, at least 3 of them wear glasses.

(b) Given that there are exactly 4 students in the tutorial classes wearing glasses, find the probability that each class has at least one student wearing glasses.

(5 marks)

9. In a city, $\dfrac{9}{20}$ of the citizens are males. $\dfrac{1}{5}$ of the male citizens and $\dfrac{2}{5}$ of the female citizens have participated in voluntary work before. 8 citizens of the city are interviewed randomly and independently.

(a) Find the probability that exactly 5 interviewees have participated in voluntary work before.

(b) Given that all 8 interviewees have participated in voluntary work before, find the probability that exactly 5 of them are males.

(c) Given that all 8 interviewees have not participated in voluntary work before, find the probability that exactly 5 of them are males.

(9 marks)

10. An ice cream producer manufactures 3 types of ice cream cones A, B and C in the ratio 1 : 3 : 4. It was reported that some ice cream cones produced on 1 September 2019 were contaminated. All ice cream cones produced on that day were withdrawn and a test was carried out. The test result showed that 45% of ice cream cone A, 42% of ice cream cone B and 40% of ice cream cone C were contaminated.
    (a) An ice cream cone produced on that day is selected randomly.
        (i) Find the probability that the ice cream cone is not contaminated.
        (ii) Given that the ice cream cone selected is contaminated, find the probability that it is an ice cream cone B.
    (b) 10 ice cream cones produced on that day are selected randomly. Find the probability that at least 3 of them are not contaminated.

(6 marks)

11. Let k be a constant.
    (a) (i) Expand $e^{2kx} + e^{-2kx}$ in ascending powers of x as far as the term in x^2.
        (ii) Hence, or otherwise, expand $(e^{2kx} + e^{-2kx})^2$ in ascending powers of x as far as the term x^2.
    (b) If the sum of the coefficients of x and x^2 in the expansion of $(1-kx)^6(e^{2kx} + e^{-2kx})^2$ is 52, find the value(s) of k.

(6 marks)

12. Consider the curve C: $y = x\sqrt{x^2+1} + 3x$.
    (a) Find $\frac{dy}{dx}$.
    (b) Two of the tangents to C are parallel to the straight line $26x - 3y + 7 = 0$. Find the equations of the two tangents.

(6 marks)

Section B (25 marks)

13. The number of vehicles entering a car park in an hour follows a Poisson distribution with a mean of 5.6. There are different parking zones in the car park and each parking zone can only accommodate 4 vehicles.
    (a) Find the probability that a parking zone will be full in a certain hour.

    (2 marks)

    (b) If the first parking zone is full in a certain hour, a second parking zone will be opened immediately; and if the second parking zone is full again in that hour, a third parking zone will be opened, and so on. Find the probability that the third parking zone is opened in a certain hour but not for the fourth parking zone.

    (2 marks)

    (c) It is known that 80% of the vehicles entering the car park are private cars.
        (i) Find the probability that in a certain hour, the third parking zone is opened but not for the fourth parking zone and there are exactly 4 private cars in each parking zone.
        (ii) Find the probability that in a certain hour, the third parking zone is opened but not for the fourth parking zone and there are exactly 3 private cars in each parking zone.
        (iii) Given that in a certain hour, the third parking zone is opened but not for the fourth parking zone, find the probability that there are at least 3 private cars in each parking zone.

    (8 marks)

14. A research on the number of crabs in a pond under different temperature conditions is conducted. The temperature P (in °C) of the pond can be modelled by

$$e^P = \frac{1}{a}e^{kt},$$

where a and k are constants and t $(0 \le t \le 24)$ is the number of months since the start of the research.

(a) (i) Express P as a linear function of t.

(ii) It is given that the slope and the intercept on the vertical axis of the graph of this linear function of t are 0.5ln 2 and –ln 4 respectively. Find the values of a and k.

(3 marks)

(b) It is given that $P = \ln\left(\frac{20-x}{x}\right)$, where x is the number of crabs (in thousand) in the pond.

(i) Show that $x = \frac{20}{2^{0.5t-2}+1}$.

(ii) Show that the function $x = \frac{20}{2^{0.5t-2}+1}$ is decreasing.

(iii) Find the maximum number of crabs in the pond during the research.

(iv) Find $\frac{d^2x}{dt^2}$. Hence, describe how $\frac{dx}{dt}$ varies during the first 24 months of the research. Explain your answer.

(10 marks)

End of Paper